DECODING OF REED-MULLER CODES

ABSTRACT

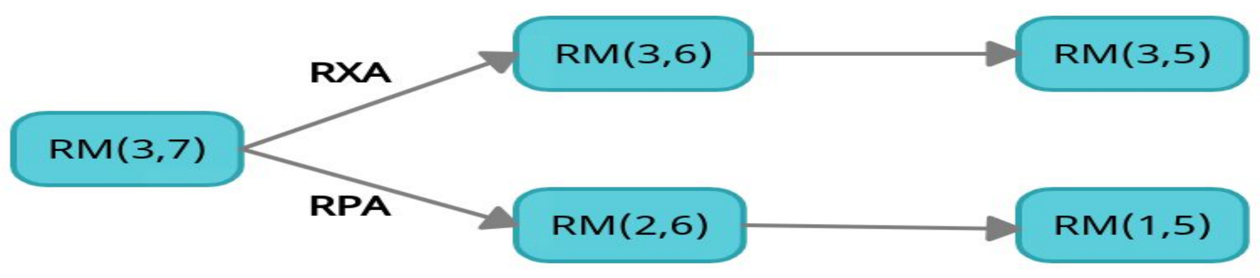
Reed-Muller Codes are among the oldest known error-correcting codes. Various algorithms for the decoding of Reed-Muller Codes have been proposed. Algorithms based on recursive projection, puncturing and syndrome decoding have been discussed below. These algorithms exploit the large symmetry group of the Reed-Muller Codes for efficiently decoding them.

DECODING ALGORITHMS

The Reed-Muller code with parameters q, m, r, denoted RM(q, m, r), is the set of evaluations of all m-variate polynomials in $F_q[X_1, \dots, X_m]$ of total degree at most r and individual degree at most q − 1 over all points in F_q.

1. **Recursive Projection-Aggregation (RPA)**

- The RPA algorithm itself consists of two steps which are performed recursively: one is a projection step and the second is an aggregation step.
- In the projection step, the noisy version of a codeword of RM (m, r) is projected onto $2^m - 1$ distinct one dimensional subspaces of the vector space F_2^m .
- After projection, the resultant vector is a corrupted version of a codeword of RM (m − 1, r − 1) code.
- The procedure of projection is repeated until first order RM code is obtained.The first order RM code is decoded using a FHT (Fast Hadamard Transform) decoder.
- The aggregation step in each iteration of the recursion involves obtaining estimate of each coordinate of the codeword by taking a majority vote based on the estimated projected codewords.
- The computational complexity of the algorithm is of the order of $n^r \log n$ and performs well on low-order RM codes.

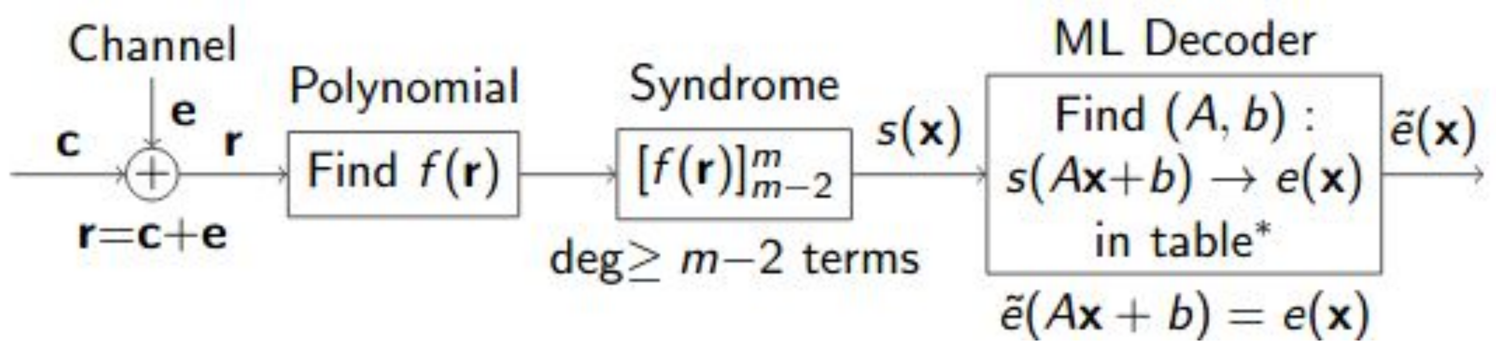


2. **Recursive Puncturing-Aggregation (RXA)**

- To extend the above idea to high-rate RM codes, RXA algorithm was proposed which involves construction of factor graphs for RM codes.
- The algorithm consists of two steps which are performed recursively : One is the puncturing step and the second is an aggregation step which is similar to that of RPA.
- In the puncturing step, the noisy version of a codeword of RM(m, r) is punctured to obtain $2(2^m\text{-}1)$ vectors using the $(2^m\text{-}1)$ permutations from the automorphism group.
- After puncturing, the resultant vector is a codeword of RM(m-1, r) code. The base code here is RM(r+2, r) which is decoded using FHT type decoder.

3. **Efficient ML Decoding of RM(m , m-3) codes:**

- In this decoding algorithm, code symmetry is used to reduce the size of the syndrome table of RM(m , m-3) code.
- There are two main steps in the algorithm: First step is to develop a reduced syndrome table.
- The second step is to develop an algorithm to find an affine transformation for a given received vector so that, the syndrome of the transformed received vector is present in the reduced syndrome table.
- The computational complexity of the algorithm is of the order of m^3.



WORK IN PROGRESS..

Extending the idea to decode RM(m, m-4) codes:

We are currently working on extending the above discussed symmetry based syndrome decoding approach to decode RM(m , m-4) codes by extending the syndrome table and by introducing new affine transformations.

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