



Are you game for Game Theory?

Why Are Our Cities Dirty Even though Everyone Likes It Clean?



Tragedy of Commons

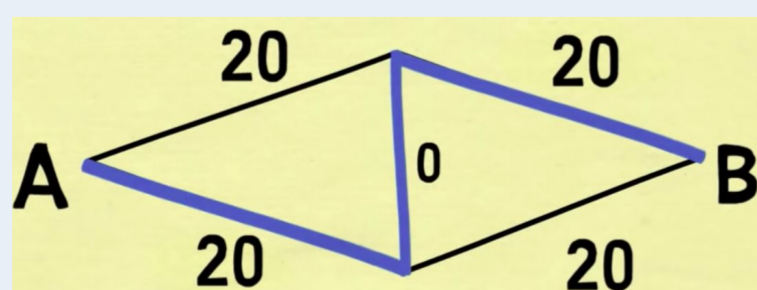
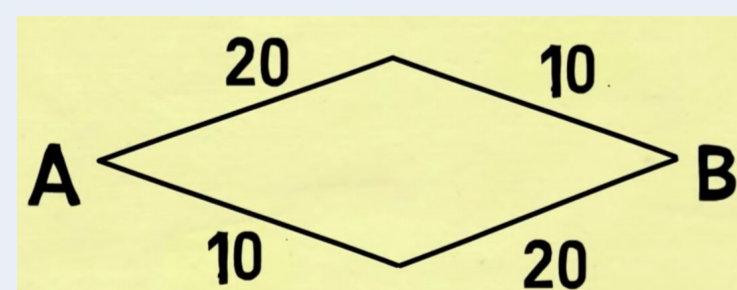
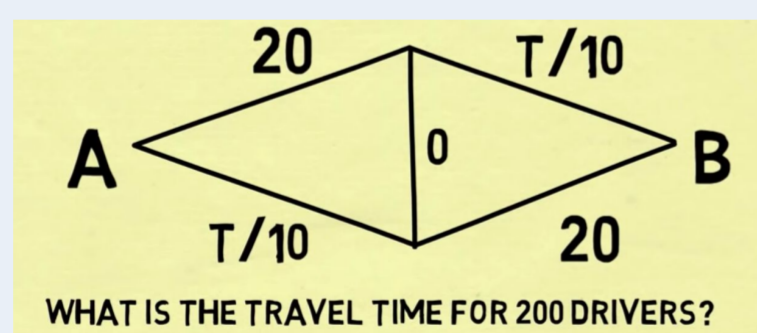
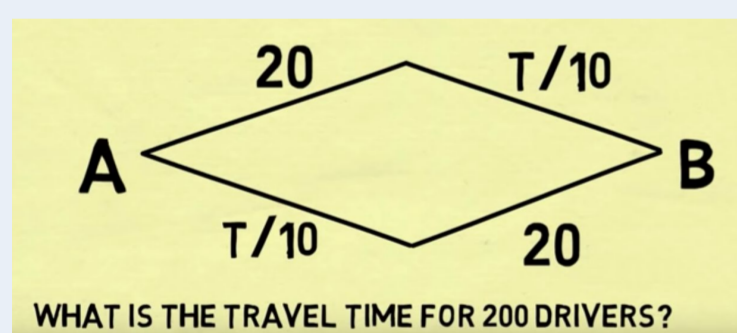
The marginal utility gained by an individual against his efforts to achieve common good is low. Such rationality leads to socially disadvantageous outcome

- Consider 2 players, A and B, trying to keep a surrounding clean
- A utility of 50 is achieved by BOTH players for every player that keeps the surrounding clean
- A player incurs a utility of -60 in his efforts to keep surrounding clean

	B keeps it clean	B keeps it dirty
A keeps it clean	40,40	-10,50
A keeps it dirty	50,-10	0,0

- When both A and B acts in self-interest, it leads to a dirty city while it would have been better off for both if they kept it clean

Do More Roads Always Mean Lesser Traffic Congestion? NO



Adding a high link road as shown increases the overall commute time for the 200 drivers travelling from A-B, as their individually rational choice leads to a socially disadvantageous outcome. This is called the **Braess' Paradox**

GAME THEORY

Models the conflicts and cooperation between rational and intelligent agents

Rational and Intelligent



Utilities and Strategies

Prisoners' dilemma

	prisoner B confess	prisoner B remain silent
prisoner A confess	5 years, 5 years	0 year, 20 years
prisoner A remain silent	20 years, 0 year	1 year, 1 year

- Agents A and B
- Utilities = - time spent in prison (-5, -5), (0, -20), (-20, 0), (-1, -1)
- Strategies = {(C, C), (C, RS), (RS, C), (RS, RS)}

What should A and B do?

1. B confesses

A will confess

2. B remains silent

A will confess

Both 1 and 2 have same answer
Confess is a Dominant Strategy

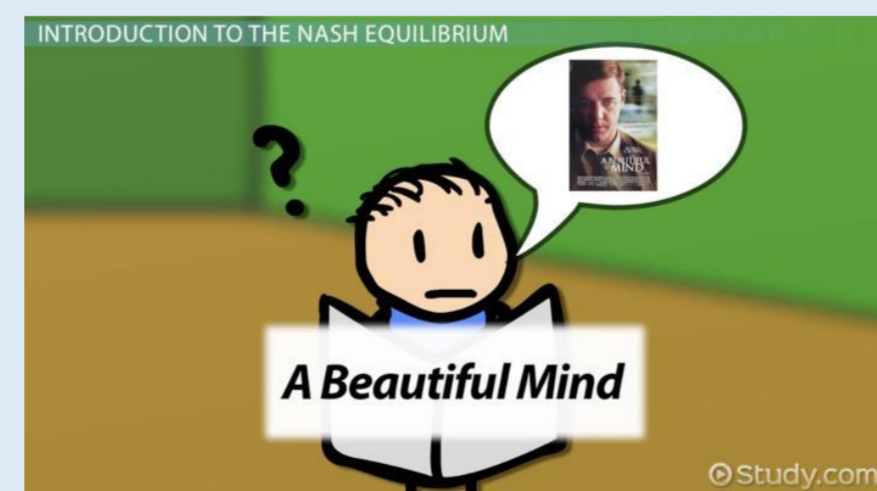
Similarly for B. Hence (C,C) is
Dominant Strategy Equilibrium

	2	
1	A	B
A	2, 1	0, 0
B	0, 0	1, 2

No Dominant Strategies !!
So which strategies would the players choose?

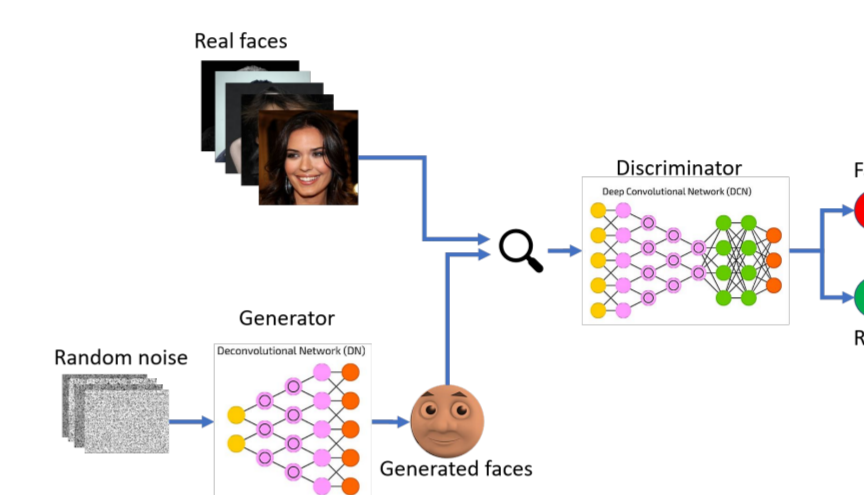
NASH EQUILIBRIUM
No incentive to deviate from (C, C) or {(A,A) (B, B)}

Will There Always be Nash equilibrium ?
(-1,-1) (-1, 1)
(-1, 1) (1, -1)



Game Theory to the rescue in complex scenarios !!

Two player zero sum games with infinite strategy space



- Players - Generator (G) and Discriminator (D)
- Strategies - Weights of D and G (Can we write it as a matrix ??)

EQUILIBRIUM !!

Or
What are the optimal weights?

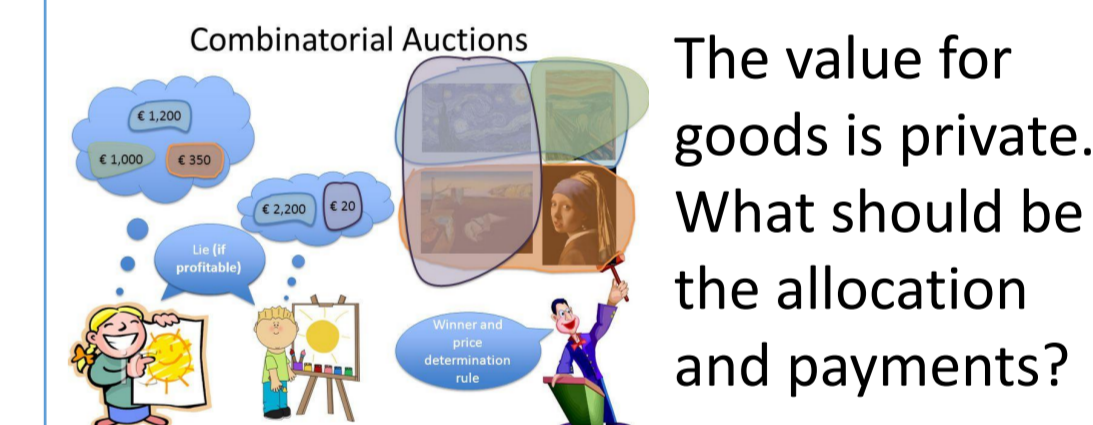
How do we find those weights in finite time? (PPAD Complete)

Incomplete Information (Bayesian Games)

Firms A and B in market. A wants to renovate. Private knowledge {High, Low investment}. B wants to enter the market

	High-investment cost		Low-investment cost	
	Enter [y]	Refrain [1-y]	Enter [x]	Refrain [1-x]
Modernize	0, -2	4, 0	3, -2	7, 0
Status quo	4, 2	6, 0	4, 2	6, 0

Sealed-bid auctions



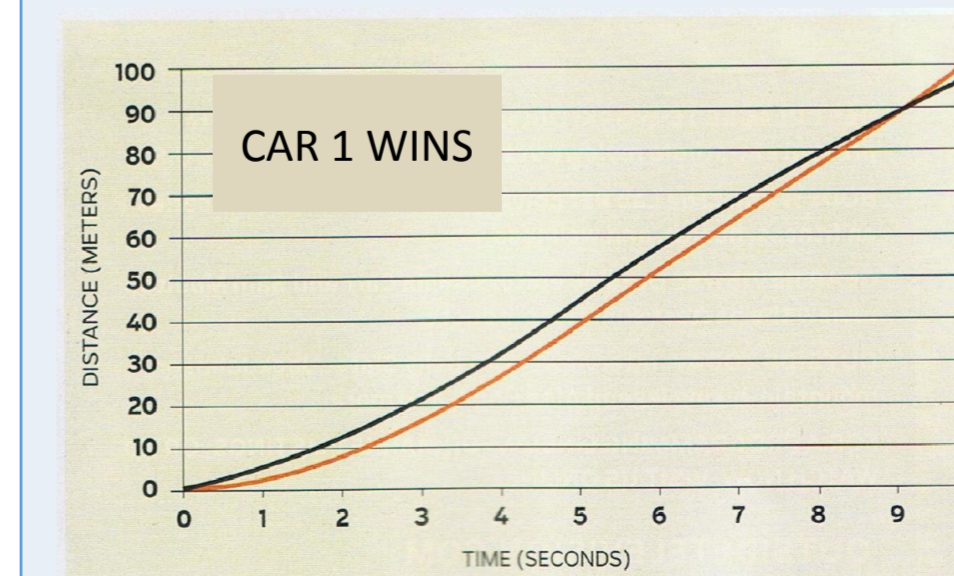
The value for goods is private. What should be the allocation and payments?

Differential Games

(Optimal Control + Game Theory)

CAR 1 (red) vs CAR 2 (blue)

Control Variables Accelerator pedal positions
Fuel Consumption Rates: $y_1(t), y_2(t)$
Fuel Cost: p_1, p_2
 $c = (c_1(t), c_2(t))$
Price Money: M, Total Time: T
Positions: $r_1(t), s_1(t)$
 $r_2 = r_1'(t), s_2 = s_1'(t)$
State Variables
 $z = (r_1, r_2, s_1, s_2)$
 $z' = f(t, z, c)$



Distance Vs Time, straight line for first player

Payoff for player 1

$$M + \int_0^T (-y_1 * p_1) dt$$

Applications
Economics, Military, etc

