INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY
HYDERABAD

R&D SHOWCASE 2021
Technology, Social Impact

Witnessing Negative Conditional Entropy

Objective

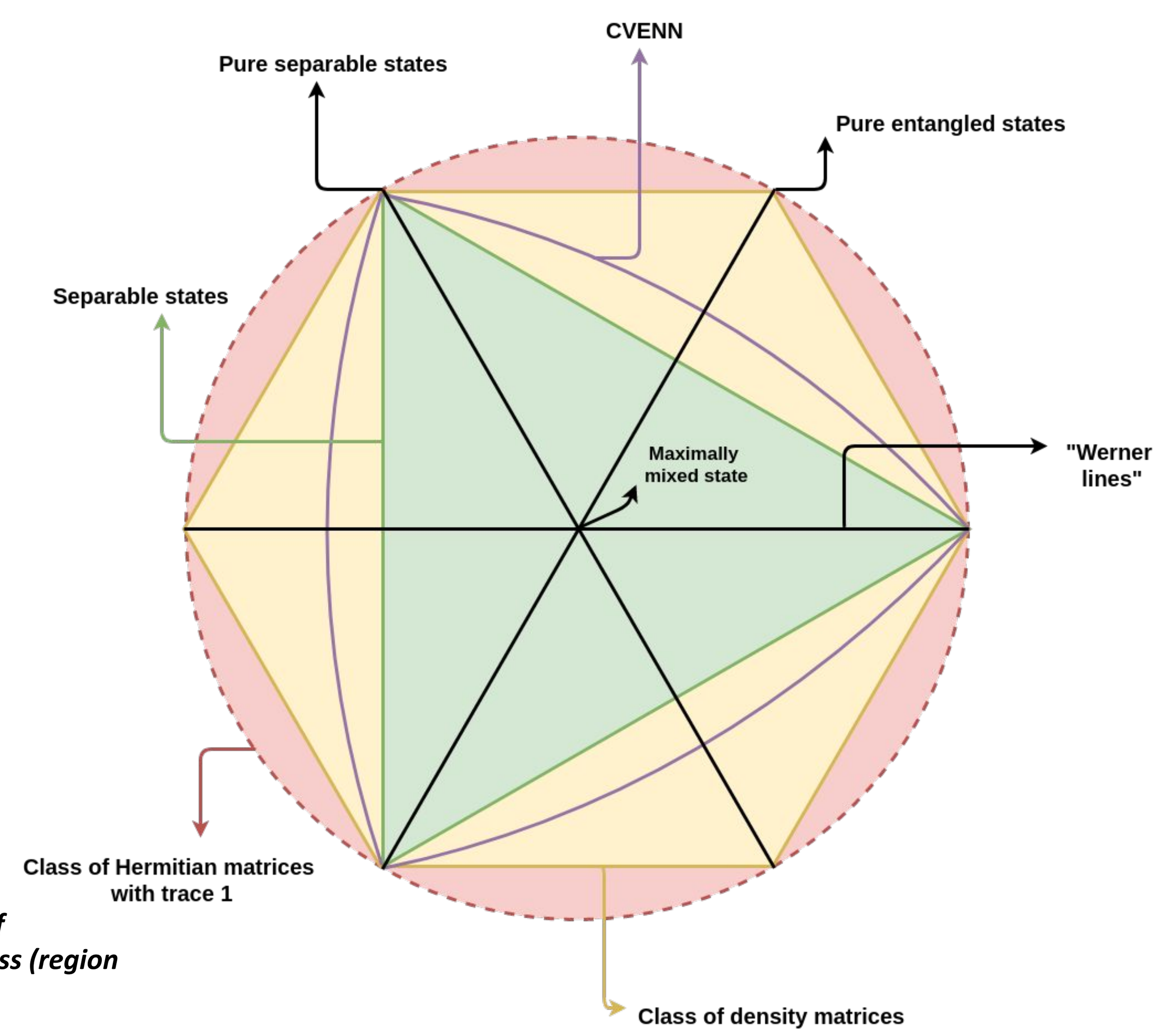
- To characterize the class of bipartite quantum states of dimensions $d \otimes d$ that have non-negative quantum conditional entropy (QCE).
- To design a witness for states with negative QCE.
- To give an upper bound to the QCE of an unknown state.

Background

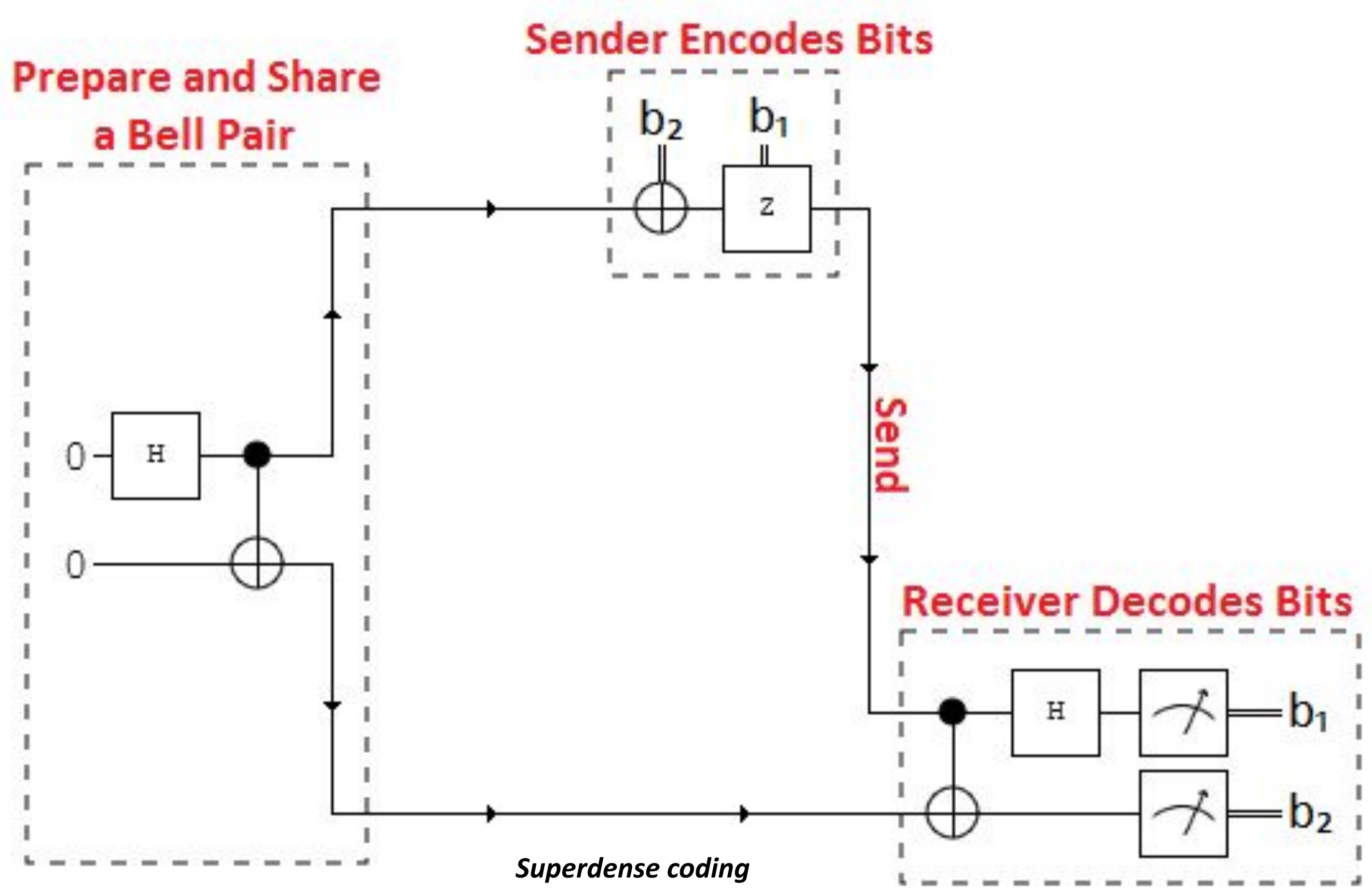
- Quantum conditional entropy (QCE) is a property of bipartite quantum states. It is defined as the difference between the von Neumann entropy of the whole system and the von Neumann entropy of a subsystem:

$$S_{A|B}(\rho_{AB}) = S(\rho_{AB}) - S(\rho_B)$$

- States with negative QCE are useful for several information theoretic tasks (see Applications).
- Hence, there is a need to classify states to understand whether they are useful for the tasks, and the extent to which they are, i.e. a resource theory is essential.
- A witness for negative QCE is an operator whose expectation value is non-negative in all states with non-negative QCE, and negative in atleast one state with negative QCE.
- Witnesses are useful in laboratory experiments in the detection of resource (here, negative QCE)



Schematic diagram of non-negative QCE class (region inside purple figure)



Superdense coding

Methods

- We use results from Functional Analysis: the Heine-Borel theorem and the Hahn-Banach theorem to characterize the class of states with non-negative QCE and to prove the existence of a witness.
- To construct the witness, we use a geometric derivation as well as a result from quantum information - the monotonicity of relative entropy:

$$S(\sigma_B||\rho_B) \leq S(\sigma_{AB}||\rho_{AB})$$

Results

- The class of states with non-negative QCE is convex and compact.
- An analytical witness for the class detecting a state ρ_{AB} is:

$$W = -log(\rho_{AB}) + I \otimes log(\rho_B)$$

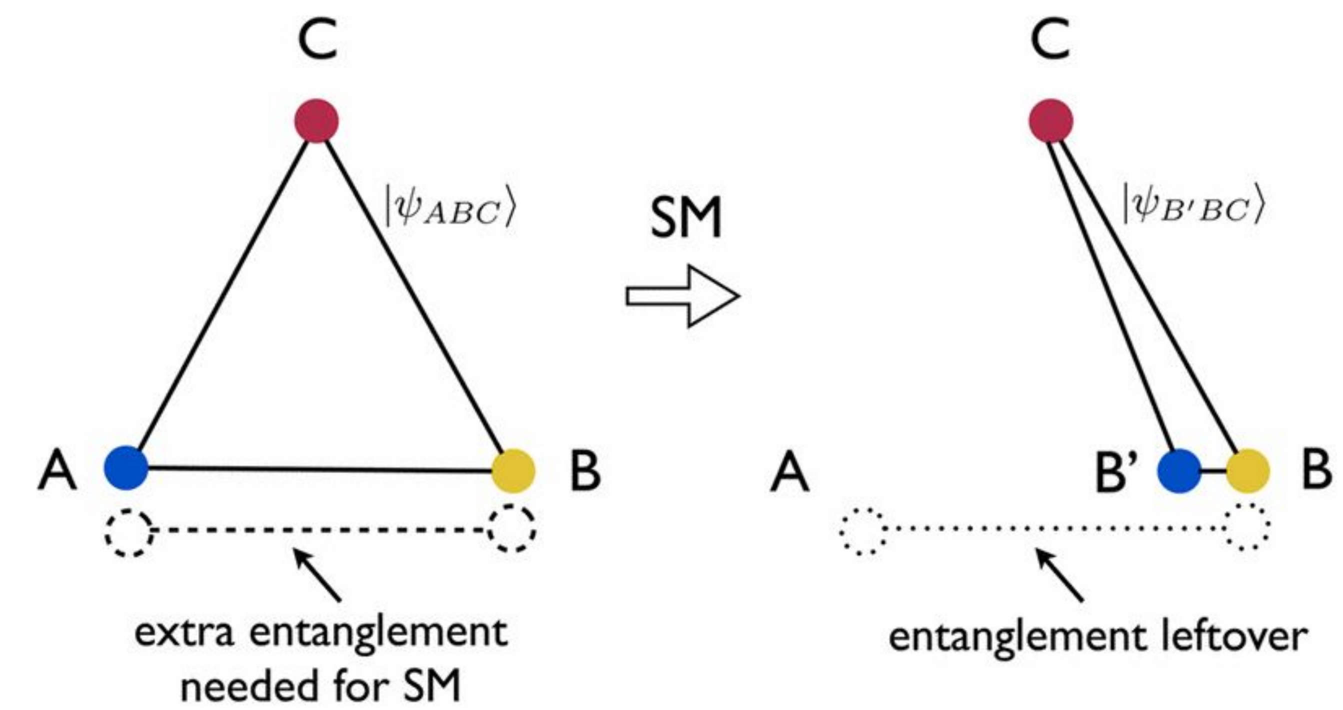
- The expectation value of the above witness in a state serves as an upper bound to the QCE of the state.
- A geometric witness for the class is:

$$W_g = \frac{Tr(\sigma_c\rho_s - \sigma_c^2)I + \sigma_c - \rho_s}{\sqrt{Tr(\sigma_c - \rho_s)^2}}$$

where ρ_s is the state that is separated and σ_c is the state within the class that is closest to ρ_s.

Applications

- **Superdense coding** is the ability to communicate more information using a quantum system (provided some resource is shared beforehand) than via a classical system of the same dimensions. For example, using a single qubit to send 2 classical bits worth of information. States with negative QCE are useful for superdense coding.
- **Quantum state merging** is the complete transfer of a shared state between two parties to one party. If the two parties share a state with negative QCE, then not only does the process of state merging become trivial, but there is also leftover entanglement shared between the two parties that can be used as a resource for further transfer of quantum states.



State merging

- **Reduction in Measurement Uncertainty:** Consider the following experiment:
  - Alice and Bob agree on two measurements X and Y.
  - Bob prepares a state and sends it to Alice who randomly measures X or Y and reveals the measurement and outcome.
  - Bob's aim is to prepare the initial state such that he reduces the uncertainty about the outcome revealed by Alice. This is bounded by the inequality: $H(X) + H(Y) \geq log_2 \frac{1}{c}$
  - If Bob has access to quantum memory, he can send Alice a state that is correlated with his memory. Then the new inequality is:

$$H_A(X|B) + H_A(Y|B) \geq log_2 \frac{1}{c} + S_{A|B}(\rho_{AB})$$

  - Therefore, if a state with negative CVE is shared, it beats the original uncertainty bound. ●

Authors: *Mahathi Vempati, Nirman Ganguly, Indranil Chakrabarty, Arun K. Pati* **Research Centre:** *CSTAR, IIIT-H* **Presenter email:** *mahathi.vempati@research.iiit.ac.in*
Image Credits: *Superdense coding: Strilanc, State merging: Aharon Brodutch* **Link to paper:** *https://arxiv.org/abs/2001.11237*