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A Minimal Neuronal Model for Synaptic Integration

ABSTRACT

The FitzHugh-Nagumo (FN) system is a prototypical model for simulating different modes of neuronal spiking activity. We propose a simple formulation of the FN model to simulate the phenomenon of synaptic integration, i.e., the processing of synaptic action potentials to generate postsynaptic spikes. We further show how the model can be used to probe the dynamical properties of postsynaptic spike trains, such as their responses to noise. We consider the phenomena of stochastic and coherence resonance to illustrate the same.

METHOD

- The FitzHugh-Nagumo equations consist of two state variables whose dynamics operates on significantly different timescales.
- The fast "voltage" variable ***(v)*** mimics a neuron's membrane potential and is complemented by a slow "recovery" variable ***(w)***.

$$\boxed{\varepsilon\frac{dv}{dt}=v(v-a)(1-v)-w+I(t)+\sigma\xi(t)}$$

$$\boxed{\frac{dw}{dt}=v-w-b}$$

- The function ***I(t)*** models a time dependent current injection into the neuron and with appropriate choices of ***I(t)***, the neuron can be driven to fire an action potential.
- Stochastic perturbations to the membrane potential are modelled using white noise $\boldsymbol{\xi(t)}$ with noise intensity (standard deviation) $\boldsymbol{\sigma}$.
- To extend the FN equations to describe a postsynaptic neuron, we take a minimal approach and model synaptic inputs in terms of rectangular current pulses. The current term for the postsynaptic neuron is thus formulated:

$$\boxed{I_{post}(t)=\frac{I_T}{N_{min}}\sum_{i=1}^{N}H(v_i-v_T)}$$

- The membrane potential of each presynaptic neuron, v_i , is converted to a rectangular pulse whenever it exceeds a threshold value, v_T .
- The parameter I_T can be interpreted as the firing threshold for the summation of all presynaptic input pulses.
- A coupling parameter is introduced via N_{min} which represents the minimum number of synaptic inputs that need to overlap in order to elicit postsynaptic firing.

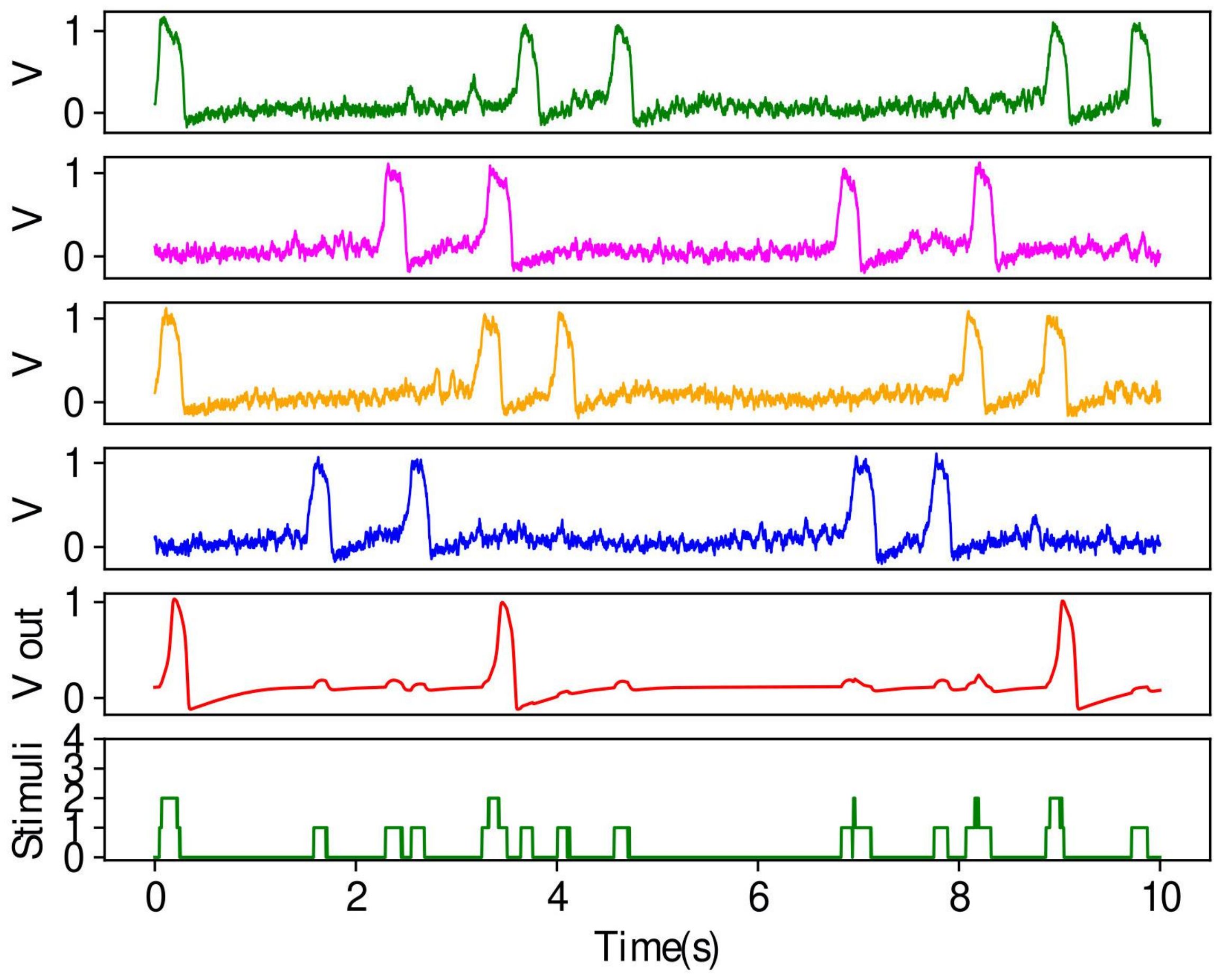

Fig. 1

RESULTS

- Figure 1 shows sample trajectories of 4 noisy presynaptic neurons stimulated by a sinusoidal current. The postsynaptic response (noiseless) is shown(red, row 4) along with the summation of rectangular input pulses (green, row 5). Our model supports spatial and temporal summation of synaptic inputs that are observed for biological neurons [1].

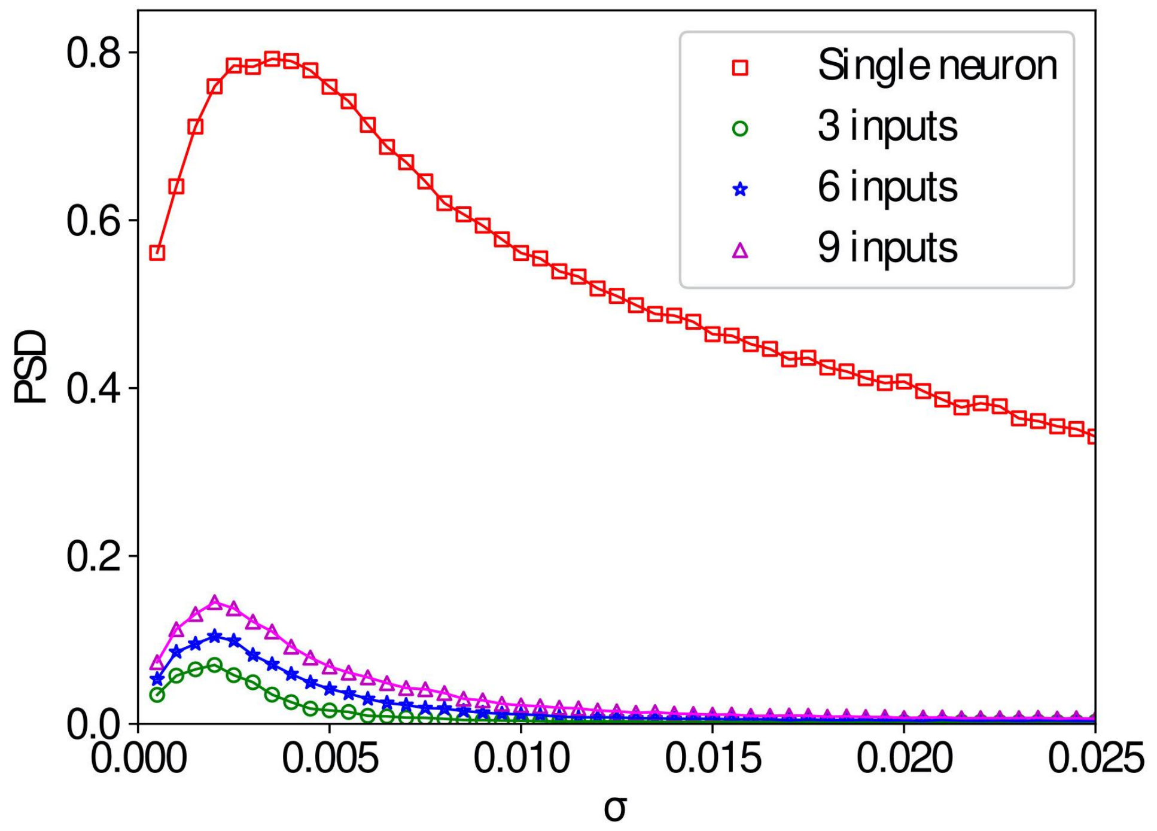

Fig. 2a

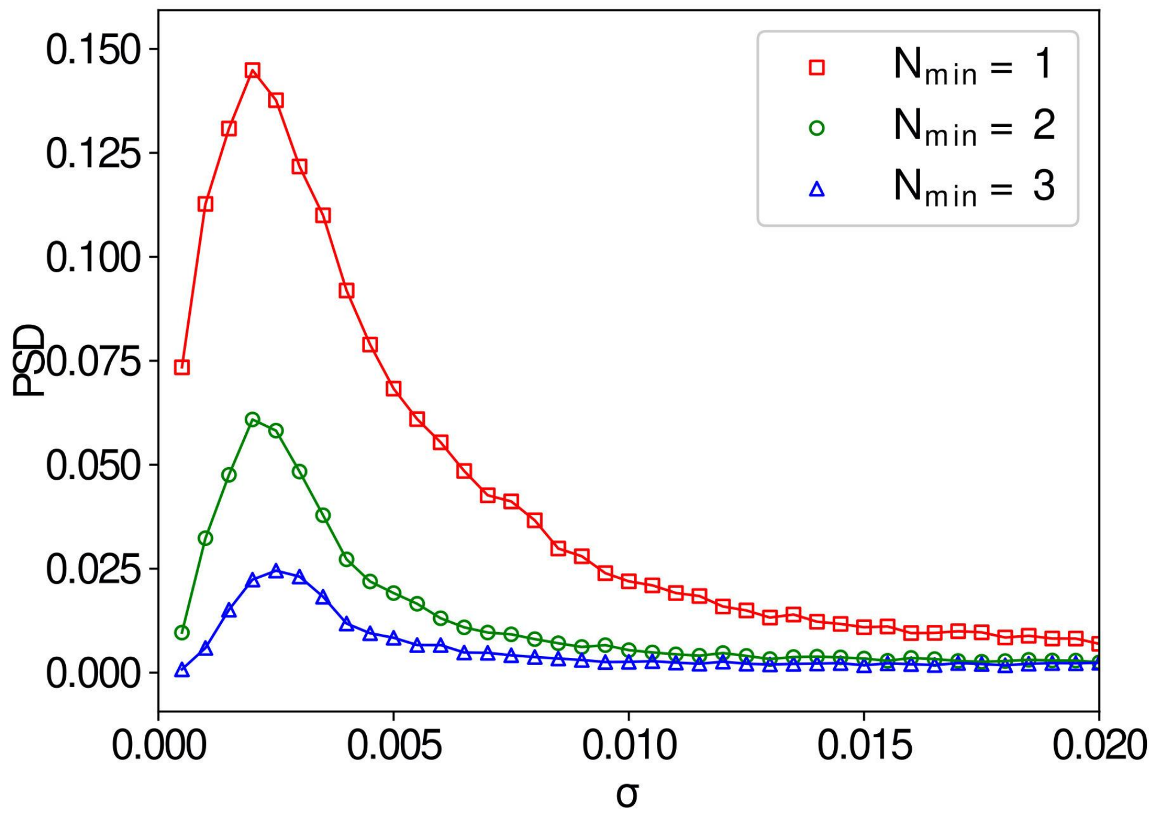

Fig. 2b

- Stochastic resonance is a noise driven phenomenon in which the addition of an optimal amount of white noise to a sinusoidal input current leads to maximum synchronization between the input signal and output neuronal spikes.
- The effect can be shown by plotting the power spectral density (PSD) of the neuronal response at the driving frequency (0.2 Hz in Fig. 2) versus the noise intensity σ.
- In figure 2a, we show stochastic resonance for a presynaptic neuron, and postsynaptic neurons with different number of synapses. The parameter $\mathbf{N_{min}}$ is set to 1.
- In figure 2b, we show stochastic resonance for a postsynaptic neuron with 9 inputs for different values of the coupling parameter, $\mathbf{N_{min}}$.
- The PSD peak for postsynaptic neurons with few synapses is lower compared to presynaptic neurons but occurs earlier.
- The smaller value of optimal noise intensity for postsynaptic neurons could have a functional role in the structure and connectivity of sensory/peripheral neurons [1].
- Decreasing the coupling by increasing $\mathbf{N_{min}}$ lowers the PSD peak as shown in Fig. 2b.
- In the coherence resonance effect, neurons are purely stimulated by white noise input currents.
- At an optimal noise intensity, the output spike trains are most coherent (decay of the autocorrelation function is the slowest). A widely used measure of coherence is the coefficient of variation (CV) of inter-spike intervals which hits a minimum at the optimal noise intensity.
- As shown in Fig. 3, postsynaptic neurons are more coherent than their presynaptic counterparts and hit the point of maximum coherence earlier.
- This suggests a possible link to the existence of spontaneous mean firing rates of different classes of neurons and the factors which may influence those rates (rate coding, coherence, etc) [1].

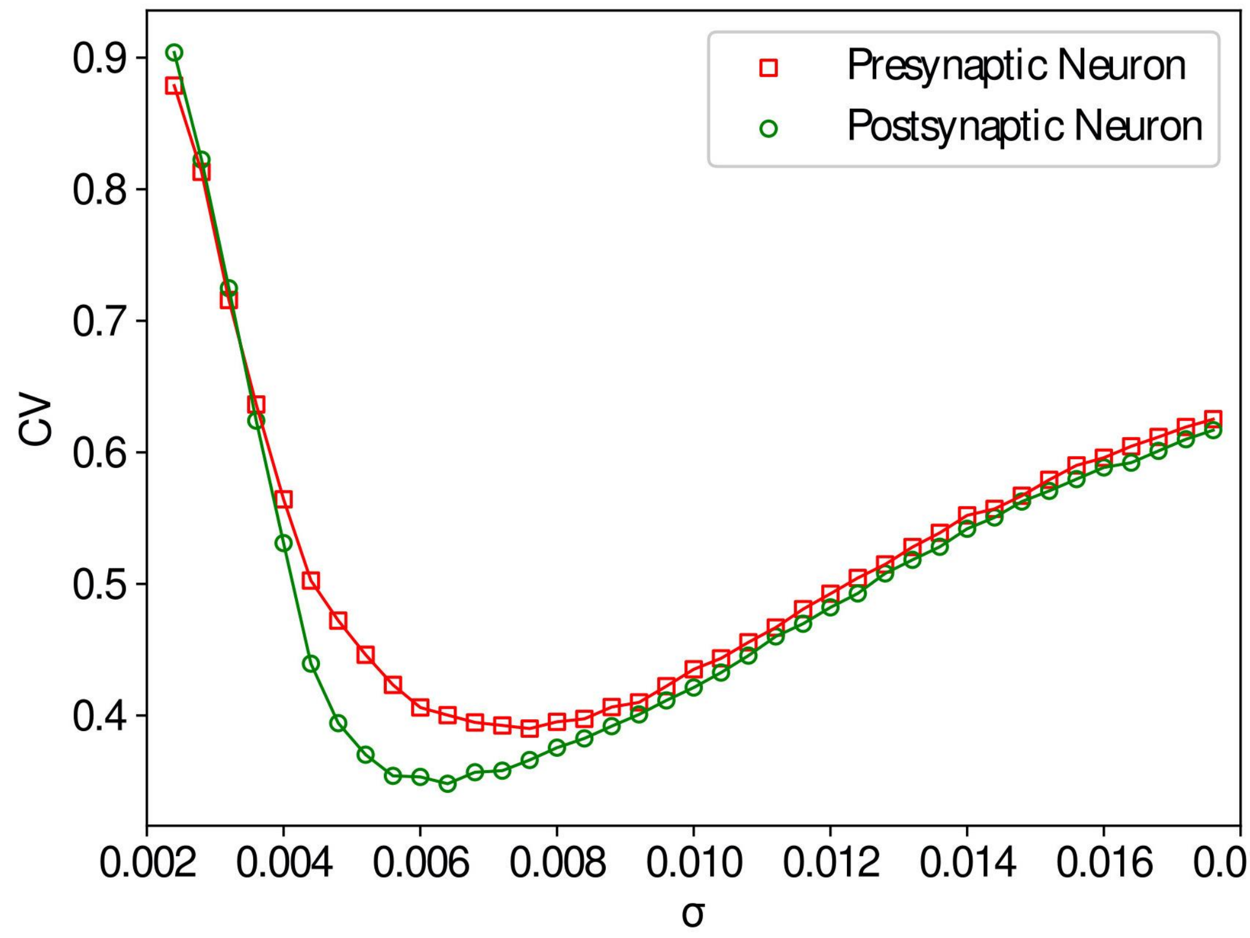

Fig. 3

[1] A. Alva and H Singh, The Fitzhugh-Nagumo model for synaptic integration, (2020) arXiv:2012.05454v2 [q-bio.NC]

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